A-REI Braking Distance

Task

The braking distance, in feet, of a car traveling at v miles per hour is given by

$$d = 2.2v + \frac{v^2}{20}.$$

a. What is the braking distance, in feet, if the car is going 30 mph? 60 mph? 90 mph?

b. Suppose that the car took 500 feet to brake. Use your computations in part (a) to make a prediction about how fast it was going when the brakes were applied.

c. Use a graph of the distance equation to determine more precisely how fast it was going when the brakes were applied, and check your answer using the quadratic formula.

IM Commentary

The purpose of this task is to give an application arising from a real-world situation in which a quadratic equation arises, and where it is natural to use a graphical method to find an approximate solution and the quadratic formula to find an exact solution. Notice that although the graphical method can give a good approximation for the particular value $d = 500$ chosen here, the quadratic formula is necessary for expressing v as a function of d in general.

Solution

a.

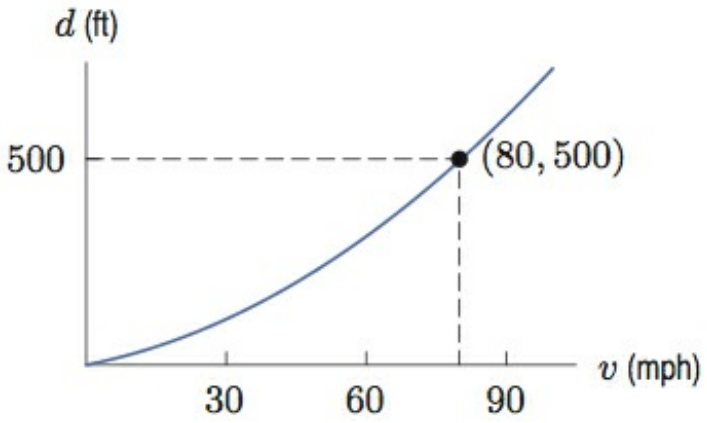
$$\text{At 30 mph, braking distance } = d = 2.2(30) + \frac{30^2}{20} = 111 \text{ ft.}$$

$$\text{At 60 mph, braking distance } = d = 2.2(60) + \frac{60^2}{20} = 312 \text{ ft.}$$

$$\text{At 90 mph, braking distance } = d = 2.2(90) + \frac{90^2}{20} = 603 \text{ ft.}$$

b. From our answers in part (a), we see that the speed for a braking distance of 500 ft is between 60 mph and 90 mph. Estimates within this range can vary, though based on the computations, it is reasonable to predict a velocity closer to 90 than to 60.

c. The figure shows a graph of $d = 2.2v + v^2/20$. We see that the speed corresponding to a braking distance of 500 ft is approximately 80 mph.



To find the speed more accurately, we use the quadratic formula to solve the quadratic equation:

$$500 = 2.2v + \frac{v^2}{20} \text{ which is } \frac{1}{20}v^2 + 2.2v - 500 = 0,$$

giving $v = 80.39$ and $v = -124.39$. On physical grounds we want $v > 0$, so $v = 80.39$, which is in good agreement with our graphical estimate of 80 mph.