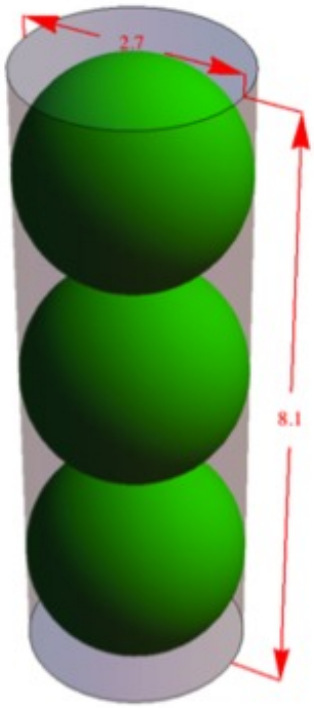
G-GMD, G-MG Tennis Balls in a Can

Task

The official diameter of a tennis ball, as defined by the International Tennis Federation, is at least 2.575 inches and at most 2.700 inches. Tennis balls are sold in cylindrical containers that contain three balls each. To model the container and the balls in it, we will assume that the balls are 2.7 inches in diameter and that the container is a cylinder the interior of which measures 2.7 inches in diameter and $3 \times 2.7 = 8.1$ inches high.



a. Lying on its side, the container passes through an X-ray scanner in an airport. If the material of the container is opaque to X-rays, what outline will appear? With what dimensinos?

b. If the material of the container is partially opaque to X-rays and the material of the balls is completely opaque to X-rays, what will the outline look like (still assuming the can is lying on its side)?

c. The *central axis* of the container is a line that passes through the centers of the top and bottom. If one cuts the container and balls by a plane passing through the central axis, what does the intersection of the plane with the container and balls look like? (The intersection is also called a *cross section*. Imagine putting the cut surface on an ink pad and then stamping a piece of paper. The stamped image is a picture of the intersection.)

d. If the can is cut by a plane parallel to the central axis, but at a distance of 1 inch from the axis, what will the intersection of this plane with the container and balls look like?

e. If the can is cut by a plane parallel to one end of the can—a horizontal plane—what are the possible appearances of the intersections?

f. A cross-section by a horizontal plane at a height of $1.35 + w$ inches from the bottom is made, with $0 < w < 1.35$ (so the bottom ball is cut). What is the area of the portion of the cross section inside the container but outside the tennis ball?

g. Suppose the can is cut by a plane parallel to the central axis but at a distance of w inches from the axis ($0 < w < 1.35$). What fractional part of the cross section of the container is inside of a tennis ball?